

# Effect of Micropolar Parameters on Elastohydrodynamic Analysis of Circular Bearing

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**Abstract-** This paper presents the effect of deformation of the bearing shell on the static and dynamic characteristics. The modified Reynolds equation and energy equation are derived on the basis of Eringen's micropolar fluid theory and are solved by finite element method. Flexibility of bearing shell is also taken into account. Static and dynamic performance characteristics are presented for a wide range of deformation coefficient and the effect of material characteristics length and coupling number on the performance are presented. An improved performance has been observed at higher coupling numbers.

**Key Words:** Micropolar, characteristics length, coupling number, Elastohydrodynamic, finite element method

## 1 INTRODUCTION

EARLIER the analysis of a journal bearing was used to be carried out with Newtonian fluid assumed the bearing to be rigid. The performance characteristics have been evaluated by many researchers [1-3] under such assumptions. However, lately, it was realized that as under heavy load, the bearing shell deforms and clearance space changes. This results in the fluid film thickness changes that affect other characteristics changes from the performance value of Newtonian fluid. In the present paper elastohydrodynamic analysis is carried out to obtain the effect of micropolar parameters on the performance of a circular bearing.

Prakash and parwal [6] applied lubrication theory for micropolar fluid and determined the static characteristics. Guha et al [7] computed the static performance characteristics of journal bearing with micropolar fluid. Guha et al [8] also worked on misaligned circular bearing. V.P.Sukumaran Nair and Prabhakaran Nair [5] worked on Elastohydrodynamic behavior of circular bearing and discussed the effect of additives concentration in lubricant. It has been observed that no work has been published so far showing the micropolarity effect on Elastohydrodynamic analysis of circular bearing.

## 2 ANALYSIS

### 2.1 Basic Equations

General form of governing equations for micropolar fluids, as per Eringen's theory [1] and [8] are as:

$$\rho \frac{d\vec{v}}{dt} = 0 \quad (1)$$

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + (\lambda + \mu - \mu_r) \nabla(\nabla \cdot \vec{v}) + (\mu + \mu_r) \Delta \vec{v} + 2\mu_r (\nabla \times \vec{\omega}) \quad (2)$$

$$\rho \frac{d\vec{\omega}}{dt} = (c_0 + c_D - c_A) \nabla(\nabla \cdot \vec{\omega}) + (c_D + c_A) \Delta \vec{\omega} + 2\mu_r (\nabla \times \vec{v} - 2\vec{\omega}) \quad (3)$$

The above equations (1) to (3) are conservation of mass, conservation of linear momentum and conservation of angular momentum respectively. In these equations,  $p$  is the pressure,  $\rho$  is the density,  $\mu$  and  $\lambda$  are dynamic viscosity coefficient and second viscosity coefficient respectively, while  $\mu_r$  represents the dynamic micro rotation viscosity.  $c_0, c_A$  and  $c_D$  are the coefficients of angular velocity. Components of velocity vector  $\vec{v}$  and microrotational velocity  $\vec{\omega}$  are given as

$$\vec{v} = (v_x, 0, v_z), \quad \vec{\omega} = (\omega_x, 0, \omega_z) \quad (4)$$

Neglecting the fluid film curvature as the fluid film is very small. Then equation (2) and (3) becomes

$$(\mu + \mu_r) \frac{\partial^2 v_x}{\partial y^2} + 2\mu_r \frac{\partial \omega_z}{\partial y} - \frac{\partial p}{\partial x} = 0 \quad (5a)$$

$$(\mu + \mu_r) \frac{\partial^2 v_z}{\partial y^2} + 2\mu_r \frac{\partial \omega_x}{\partial y} - \frac{\partial p}{\partial z} = 0 \quad (5b)$$

$$(c_D + c_A) \frac{\partial^2 \omega_x}{\partial y^2} + 2\mu_r \frac{\partial v_z}{\partial y} - 4\mu_r \omega_x = 0 \quad (5c)$$

$$(c_D + c_A) \frac{\partial^2 \omega_z}{\partial y^2} + 2\mu_r \frac{\partial v_x}{\partial y} - 4\mu_r \omega_z = 0 \quad (5d)$$

Apply following boundary conditions at bearing and journal surfaces are

$$v_x|_{y=0} = U, \quad v_z|_{y=0} = 0, \quad \omega_x|_{y=0} = \omega_z|_{y=0} = 0 \quad (6)$$

$$v_x|_{y=h} = 0, \quad v_z|_{y=h} = 0, \quad \omega_x|_{y=h} = \omega_z|_{y=h} = 0 \quad (7)$$

Velocity components can be obtained by solving equation (5a) to (5d) and applying boundary conditions (6) and (7). Now, substituting velocity components into the equation (1) and integrating across the film, the modified Reynolds equation for micropolar fluids is derived and in the form:

$$\frac{\partial}{\partial x} \left[ \frac{\psi(N, \Lambda, h)}{\mu_{av}} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{\psi(N, \Lambda, h)}{\mu_{av}} \frac{\partial p}{\partial z} \right] = 6U \frac{\partial h}{\partial x} \quad (8)$$

where

$$\psi(N, \Lambda, h) = h^3 + 12\Lambda^2 h - 6N\Lambda h^2 \coth(Nh/2\Lambda) \quad (9)$$

Where

$$\Lambda = \sqrt{\left( \frac{C_A + C_D}{4\mu} \right)}, \quad N2 = (\mu_r / \mu + \mu_r)$$

By introducing the non-dimensional quantities

$$\bar{z} = \frac{z}{L}, \quad \bar{h} = \frac{h}{c}, \quad \theta = \frac{x}{R}, \quad lm = \frac{c}{\Lambda}, \quad \bar{P} = \frac{Pc^2}{\mu UR} \quad (10)$$

Eq. (8) can be written in the non-dimensional form as

$$\frac{\partial}{\partial \theta} \left[ \frac{\psi(N, l_m, \bar{h})}{\mu} \frac{\partial \bar{P}}{\partial \theta} \right] + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[ \frac{\psi(N, l_m, \bar{h})}{\mu} \frac{\partial \bar{P}}{\partial \bar{z}} \right] = 6U \frac{\partial \bar{h}}{\partial \theta} \quad (11)$$

$$\psi(N, l_m, \bar{h}) = \bar{h}^3 + 12 \frac{\bar{h}}{l_m^2} - 6N \frac{\bar{h}^2}{l_m} \coth\left(\frac{N\bar{h}l_m}{2}\right) \quad (12)$$

Here both  $l_m$  and  $N2$  are the important characteristic parameters of micro polar fluid distinguishing a micropolar

fluid from a Newtonian fluid.  $N2$  is a dimensionless parameter called the coupling number, which couples the linear, and angular momentum equation arising the micro rotational effect of the suspended particles in the fluid.  $l_m$  represents the interaction between the micropolar fluid and the film gap and is termed as the characteristics length of the micropolar fluid. As  $\Lambda$  vanishes the micropolarity is lost and then the lubricant is considered to behave as Newtonian.

## 2.2 Analysis of Bearing Shell Deformation

Using the potential energy theorem, a set of algebraic equations is obtained in terms of nodal displacement vector  $\{\bar{d}\} = [\bar{v}_\theta, \bar{v}_r, \bar{v}_z]^T$  for the displacement field of the bearing shell.

$$[\bar{K}]\{\bar{d}\} = C_d \{\bar{F}\} \quad (13)$$

Where  $\bar{F}$ , the force vector and  $C_d$  the deformation coefficient is given by

$$C_d = \frac{\mu_o \omega}{E} \left( \frac{R}{c} \right)^3 \frac{t_h}{R} \quad (14)$$

## 3 PERFORMANCE CHARACTERISTICS

### 3.1 Load Capacity

The load capacity is given as:

$$W_1 = -\frac{2R^3 \mu_o U}{c^2} \int_0^{(L/2R)} \int_0^{2\pi} \bar{p} \cos \theta d\theta d\bar{z} \quad (15)$$

$$W_2 = -\frac{2R^3 \mu_o U}{c^2} \int_0^{(L/2R)} \int_0^{2\pi} \bar{p} \sin \theta d\theta d\bar{z}$$

For vertical load support, the following conditions are satisfied in the journal centre equilibrium position

$$\bar{W}_2 = W \quad \text{And} \quad \bar{W}_1 = 0$$

### 3.2 Friction Force

$$F_f = -\frac{2U\mu_o R^2}{c}$$

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$$\times \int_0^{(L/2R)} \int_0^{2\pi} \left[ \mu_{av} \left( \frac{1}{\sigma \bar{h}} + \frac{\bar{h}}{2\mu_{av}} \frac{\partial \bar{p}}{\partial \theta} \right) \right] d\theta d\bar{z} \quad (16)$$

Where

$$\sigma = 1 - \frac{2N}{l_m \bar{h}} \tanh \left( \frac{N \bar{h} l_m}{2} \right)$$

### 3.3 End Leakage Flow

The end leakage flow for a journal bearing is given as:

$$Q_{leak} = \frac{-cRU}{6} \int_0^{2\pi} \psi(N, l_m, \bar{h}) \frac{\partial \bar{p}}{\partial z} \Big|_{z^*(L/2R)^{d\theta}} \quad (17)$$

### 3.4 Threshold Speed

The Threshold speed at which the journal bearing system becomes unstable is found as the roots to the characteristic equation

$$\omega^4 \alpha^4 + \omega^2 (\bar{C}_{xx} + \bar{C}_{zz}) \alpha^3 + \left[ \omega^2 (\bar{K}_{xx} + \bar{K}_{zz}) + \bar{C}_{xx} \bar{C}_{zz} - \bar{C}_{xz} \bar{C}_{zx} \right] \alpha^2 + (\bar{C}_{xx} \bar{K}_{zz} + \bar{C}_{zz} \bar{K}_{xx} - \bar{C}_{xz} \bar{K}_{zx} - \bar{C}_{zx} \bar{K}_{xz}) \alpha + (\bar{K}_{xx} \bar{K}_{zz} - \bar{K}_{xz} \bar{K}_{zx}) = 0 \quad (18)$$

A bearing becomes unstable at a speed, if any root to the above equation becomes positive.

## 4 RESULTS AND DISCUSSION

Figure 1 presents the combined effect of deformation coefficient and coupling number on load carrying capacity for a fixed characteristic length (1 m). Deformation coefficient ranging is taken as 0.0-0.2 with a step of 0.05. Results are plotted for  $L = 50$  and eccentricity of 0.6. It is observed that load varies marginally on changing flexibility, as deformation coefficient increases load decreases. Load carrying capacity increases as coupling number increases.

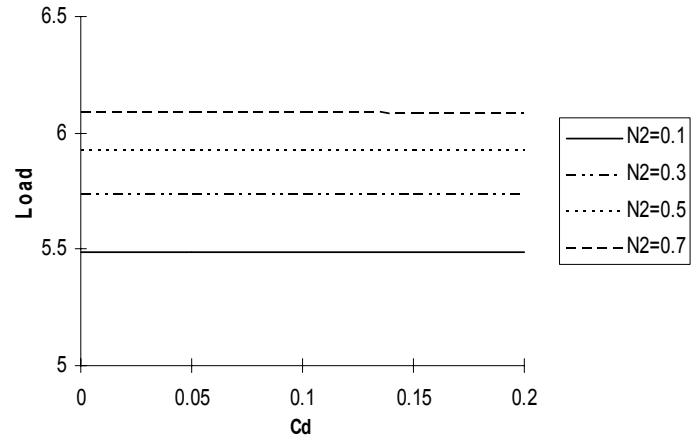
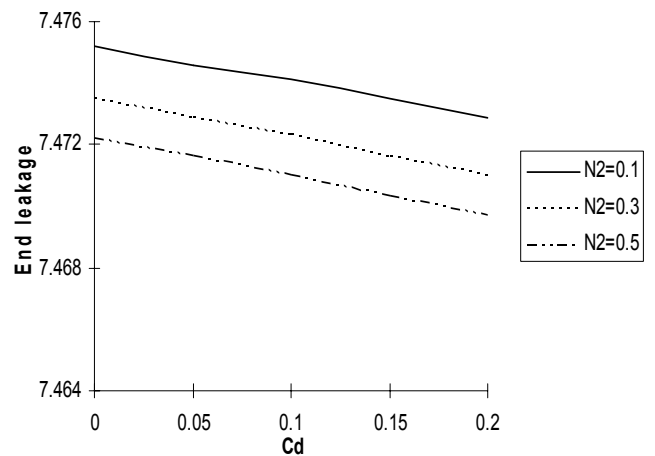
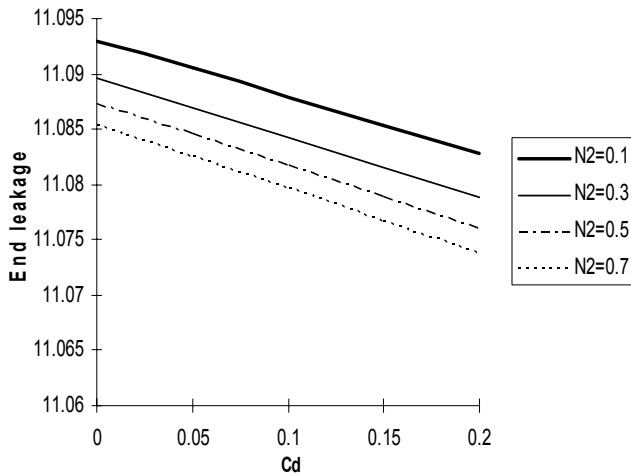


Fig.1 Variation of Load carrying capacity as a function of Cd for  $l_m=50$ , at  $\epsilon=0.6$

Figures 2(a) and (b) present the change of end leakage with the variation of deformation coefficient. It is observed that the end leakage decreases with the increase in deformation coefficient. The variation of end leakage with increase in deformation coefficient is appreciable when the bearing operates at higher eccentricity. End leakage is higher for larger coupling numbers for same characteristics length.



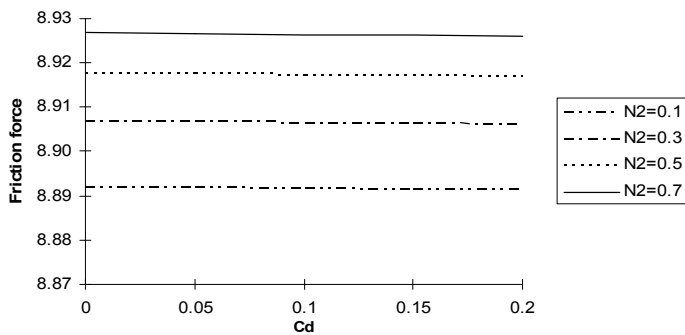
$l_m = 50$ , at  $\epsilon = 0.4$   
(a)



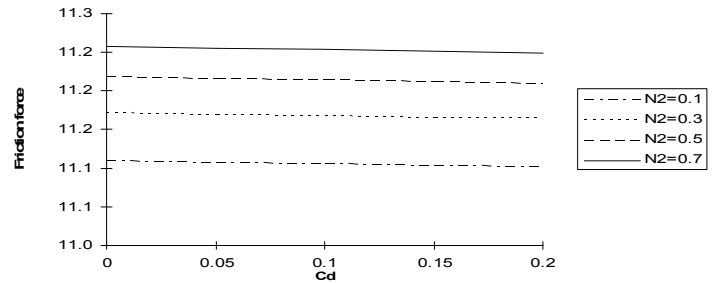
lm= 50, at ε =0.6  
(b)

Fig. 2 Variation of End leakage as a function of Cd

Fig 3(a) and (b) depicts that when bearing operates at higher eccentricity ratio large frictional losses are observed for a particular coupling number and characteristics length. Both the figures show that as the deformation coefficient increase, there is reduction in frictional losses. At lower value of deformation coefficient frictional losses are higher, while at higher value of deformation coefficient frictional losses are lower.



lm=50, at ε =0.4  
(a)



lm=50, at ε =0.6  
(b)

Fig.3 Variation of friction force as a function of Cd

Figure 4-7 depicts the variations of different stiffness with deformation coefficients. It is observed that stiffness coefficients  $K_{xx}$  and  $K_{yx}$  increases with increase in the flexibility up to a certain value of deformation coefficient and then decreases while  $K_{xy}$  and  $K_{yy}$  decreases up to a certain value. It is also observed that on increasing coupling numbers the  $K_{xx}$  and  $K_{yy}$  increases, while  $k_{xy}$  and  $K_{yx}$  decreases.

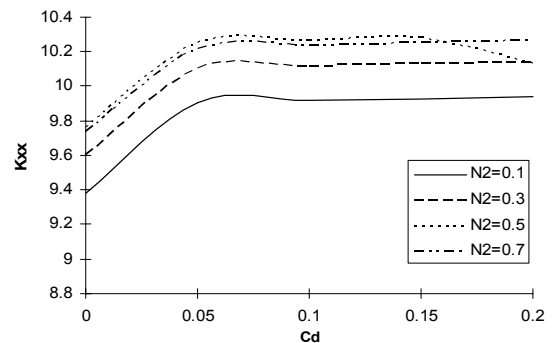


Fig. 4 Variation of Kxx as a function of Cd for lm=50, at ε =0.6

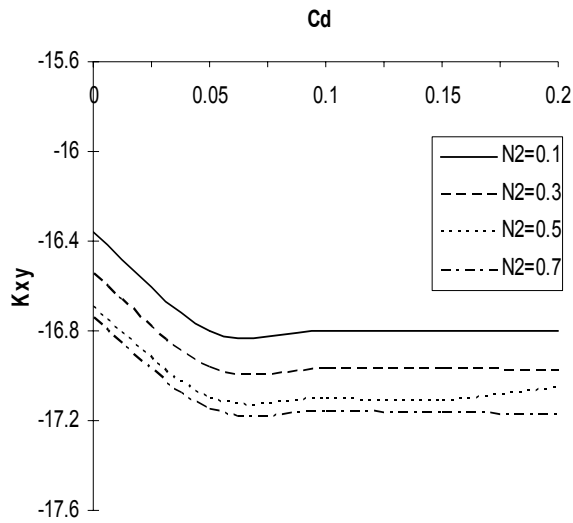


Fig. 5 Variation of  $K_{xy}$  as a function of  $C_d$  for  $Im=50$ , at  $\epsilon=0.6$

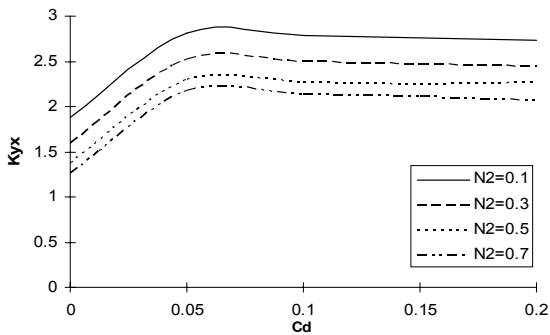


Fig. 6 Variation of  $K_{yx}$  as a function of  $C_d$  for  $Im=50$ , at  $\epsilon=0.6$

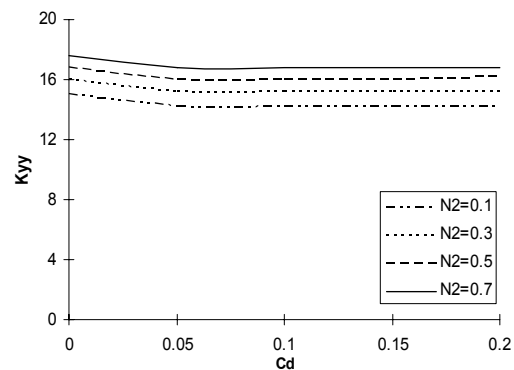


Fig. 7 Variation of  $K_{yy}$  as a function of  $C_d$  for  $Im=50$ , at  $\epsilon=0.6$

Figure 8-11 shows effect of deformation coefficient on fluid film damping coefficients. Damping coefficients  $C_{xx}$  and  $C_{yy}$  increases with deformation coefficient up to a certain value and then decreases as shown in Fig. 8 and 11. Reduction is observed in the value of  $C_{xy}$  and  $C_{yx}$  as shown in figure 9 and 10.

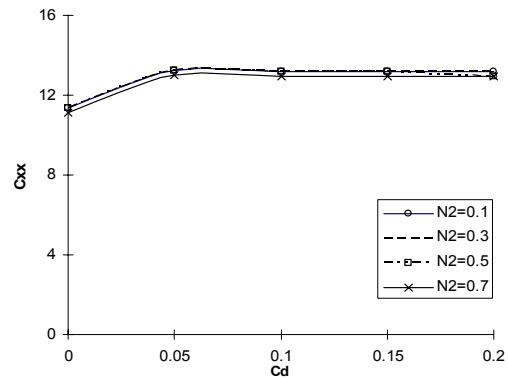


Fig. 8 Variation of  $C_{xx}$  as a function of  $C_d$  for  $Im=50$ , at  $\epsilon=0.6$

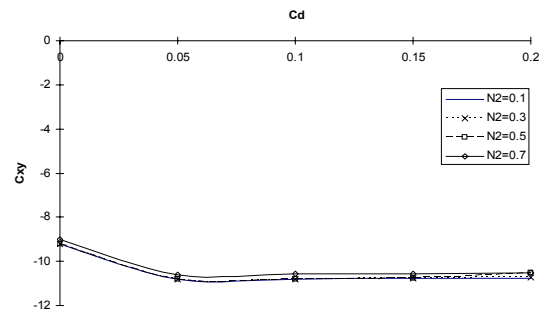


Fig. 9 Variation of  $C_{xy}$  as a function of  $C_d$  for  $Im=50$ , at  $\epsilon=0.6$

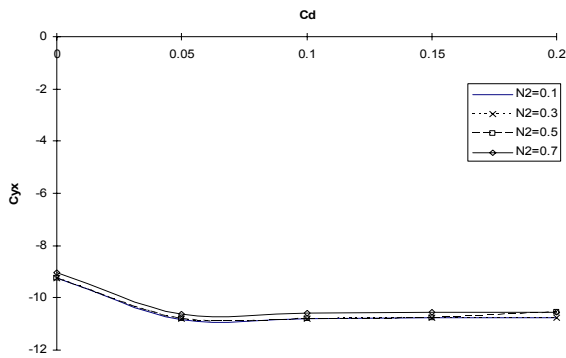


Fig.10 Variation of  $C_{yx}$  as a function of  $C_d$  for  $l_m=50$  at  $e=0.6$

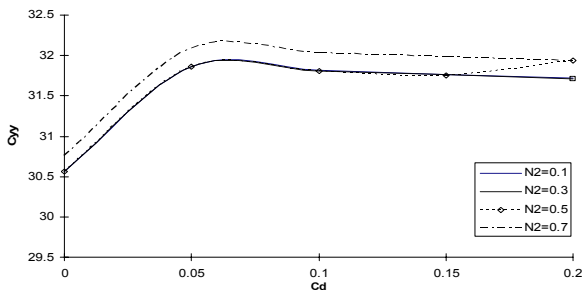
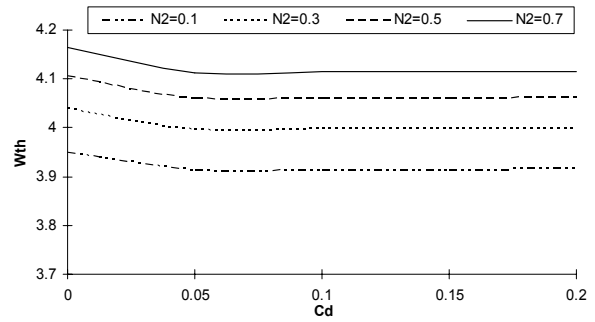
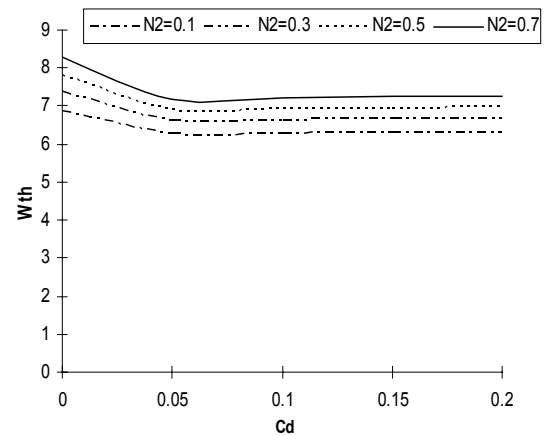


Fig.11 Variation of  $C_{yx}$  as a function of  $C_d$  for  $l_m=50$  at  $e=0.6$



$l_m=50$  at  $e=0.4$   
(a)



$l_m=50$  at  $e=0.6$   
(b)

Fig.12 Variation of  $W_{th}$  as a function of  $C_d$

Figure 12 (a) and (b) depict the variation of threshold speed with deformation coefficient. It is observed that threshold speed decreases with increase in deformation coefficient. It is also observed that threshold speed increases with increase in coupling number.

## CONCLUSIONS

1. The static parameters load capacity, end leakage, and power loss decrease when the flexibility increases.
2. Friction losses and end leakage are significantly large for larger coupling numbers for the same characteristics length and for higher eccentricity ratios.
3. Threshold speed increases for larger coupling number and significantly decreases for lower flexibility.

## NOTATIONS

$L$	Bearing width
$c$	Radius clearance
$c_0, c_A, c_D$	Coefficients of angular viscosities
$C_f$	Coefficient of friction
$E$	Bulk modulus of the Material
$F_f$	Friction force
$\bar{h}$	Film thickness
$\bar{h}$	Non-dimensional film thickness,
$J$	Microinertia constant
$l_m$	Non-dimensional characteristic length of micropolar fluid, $l_m = c / \Lambda$
$N2$	Coupling number
$P$	Hydrodynamic pressure
$\bar{P}$	Non-dimensional hydrodynamic pressure
$Q_{leak}$	End leakage flow
$R$	Journal radius
$U$	Shaft speed
$v_x, v_y, v_z,$	Components of the lubricant velocity in the x, y and z-directions, respectively
$\bar{v}_x, \bar{v}_z$	Non-dimensional components of the lubricant velocity in the x and z-directions, respectively, $\bar{v}_x = v_x / U, \bar{v}_z = v_z / U$
$W_{th}$	Threshold speed
$W_1, W_2$	Components of fluid film force in the x and z directions
$\varepsilon$	Eccentricity ratio
$\delta$	Inlet attitude angle
$\lambda$	Second viscosity coefficient
$\Lambda$	Characteristic length of micropolar fluid
$\mu$	Lubricant viscosity
$\bar{\mu}$	Non-dimensional lubricant viscosity
$\mu_0$	Inlet lubricant viscosity
$\mu_{av}$	Cross-film average value of lubricant viscosity
$\bar{\mu}_{av}$	Non-dimensional cross-film average value of lubricant viscosity
$\mu_r$	Microrotation viscosity
$\rho$	Lubricant density
$\omega_x, \omega_z$	microrotation velocity components about the axes

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